**Mathematical Modeling and Kinematics of a Glider**

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Giders are robotic vehicles used in the air and underwater to collect and transmit real-time data. Studies using gliders have important applications in oceanography, engineering, and remote sensing. The goal of this project was to model and identify aspects of a glider’s flight using vector-calculus and matrix-algebra based methods. We employed mathematical models to study the flightpath of a glider using vector valued functions and calculated the osculating plane of the glider. The model parameters were optimized to minimize turbulence. We studied the kinematics of underwater gliders using GPS data reported from gliders deployed by Rutgers University and the University of Washington. We analyzed the reported glider velocity data and applied vector-calculus based methods to calculate the instantaneous and average velocities and acceleration vectors. Additionally, we applied matrix-algebra based methods to translate and rotate the glider to position it at appropriate coordinates underwater for gathering data. This research provided insight into mathematical modeling of real-world data and involved applied optimization and data visualization. These studies provide novel avenues for hands on exploration and application of key mathematical concepts.

**Finding Finite-Dimensional Koopman Eigenfunctions for Polynomial Dynamical System**

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*Mentor: J. Nathan Kutz, Applied Mathematics*

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Systems of non-linear differential equations are often difficult to analytically solve, control, and analyze, whereas systems of linear differential equations are relatively straightforward to solve, control, and analyze because we possess tools to study systems of linear differential equations. Koopman analysis allows us to transform a system of non-linear differential equations into a linear system. The caveat is that sometimes, the resulting linear system is infinite-dimensional, meaning that the mapping between the original space and the Koopman space is infinite-dimensional. This poses a computational challenge because infinite dimensional vector spaces are difficult to computationally work with. Koopman analysis has traditionally been done on one fixed point (equilibrium). Previously, work has been done to study specific examples of systems that have closure i.e. systems with a finite-dimensional Koopman operator. In this project, we looked at how to apply Koopman operators to systems with multiple fixed points. We found effective eigenfunctions that linearize low-dimensional non-linear dynamical systems analytically if possible, computationally otherwise. Under the assumption that the right-hand side of the differential equations are polynomials, we identified appropriate eigenfunctions that linearize the Koopman space with a possible invertible mapping. Previously, we have been able to find a closed-form solution that generates eigenfunctions for one-dimensional systems that have a polynomial form. However, in practice, the resulting integral equation can be difficult to computationally solve with current methods, and edge cases such as singularities and asymptotes are not well understood. Using implicit SINDy (an algorithm that approximates dynamical systems given data), we attempted to find polynomial decompositions that allowed us to describe the eigenfunctions with rational
polynomial functions. This work is significant because better understanding dynamical systems allows us to better understand dynamic fields such as natural disaster detection, the firing of neurons, and the spread of pandemics.

**Multivariable Calculus-Based Modeling of Center of Mass and Moments of Inertia of Comet Hartley**

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Comets are cosmic snowballs of frozen gases, rock and dust that orbit the Sun. Isaac Newton suspected that comets were the origin of the life-supporting component of air and a key source for water replenishment in planetary interiors. A close-up view of comet Hartley 2 was taken by NASA’s EPOXI mission during its flyby of the comet, using the spacecraft’s medium resolution instrument. Comet Hartley has a novel asymmetric dumbbell-like shape. We employed mathematical models to study comet Hartley. Using calculus-based methods, we estimated various static properties, including the arc lengths (outer boundary length), surface area, and volume of Comet Hartley. Assuming a constant density, we also estimated the mass, center of mass, and moments of inertia for Comet Hartley using triple integration methods using cylindrical coordinates. The center of mass, moments of inertia, and radius of gyration form key inputs for studying the orbital mechanics of the comet in outer space. This research project provides excellent opportunities for hands-on explorations using multivariable calculus studies for engineering and space sciences applications. This research is important as studies of comets unravel secrets about the formation of the solar system.

**Velocity and Acceleration Profiles of Space Shuttles**

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STS-121 is a NASA space shuttle mission to the International Space Station (ISS). The ISS is a habitable satellite (Space station) in a low Earth orbit. We employ calculus-based methods to analyze and study the flightpaths, altitude, velocity, and acceleration profiles of the STS121 data reported by NASA as it travelled through outer space. Our studies unravel information about the critical points, local maxima and minima, concavity, and inflection points in the altitude data. The velocity profiles were fitted to polynomial functions using least square data fitting using linear algebra-based methods. The acceleration data involve piecewise functions which is related to the time scales involving burning of the propellent and separation of the external propellant tank as the space shuttle gets ready to move into orbit. We estimated the work done in transferring a load from Earth to the International Space station. We used optimization methods to design an optimal solar panel geometry for a satellite by minimizing the surface area. This research provides novel applications of the fundamental theorems of calculus to study motion in outer space and involves mathematical modeling, optimization, curve fitting, data analysis and data visualization.

**Goodness of Fit of Network Data**

Peter Liu, Senior, Statistics  
Mentor: Shane Lubold  
Mentor: Tyler McCormick

Network goodness of fit (GOF), which deals with determining if an observed network could have been drawn from a particular graph model, is a common method to identify network structures and a prerequisite of further network analysis. In this talk, we will present a method that constructs a test statistics using advances in Random Matrix Theory, which traces the limiting behavior of the leading eigenvalue of the observed graph’s adjacency matrix to perform network GOF. We show that under many network models - such as the Degree Corrected Stochastic Block model (DCSBM), the Beta model, and the Aggregated Relational Data (ARD) - our method performs well in identifying the correct model with high precision and low computation time. We further extend our method to the popular Latent Space model, and develop an algorithm that fast-and-accurately predicts the underlying dimension of the true model. With precision grows with network sizes, our algorithm enables better estimations for large-scale networks with minimal computational cost.

**Principal Component Analysis Reveals Political Leaning of US States Tied to Economic Status and thus Everyday Life**

Joia W Zhang, Junior, Pre-Sciences  
Mentor: Jerry Wei, Statistics  
Mentor: Abel Rodriguez, Statistics

In recent decades, partisanship between the Democratic and
Republican parties in the US has grown, resulting in congressional gridlock and economic stagnation. Untangling the relationship between partisanship and major economic factors such as household income, homeownership, population, and poverty has the potential to solve these problems. Our hypothesis is that a state’s political leaning is not associated with these factors. Our data contained the political leaning of all 50 states and DC in the 2008 presidential election, alongside 71 economic variables in the aforementioned four categories of annual household income (1984-2018), homeownership (1986-2014), population (2000-2005), and poverty rate (1990, 2000). We used principal component analysis (PCA) to condense the 71 dimensions into two dimensions for visualization, principal components (PCs) 1 and 2. They explained 48% and 13% of total variance. In the plot between the two PCs, dots represent states (and DC) colored by political leaning. The visualization revealed that a state’s political leaning is strongly tied to the 4 economic categories. Thus, we rejected our hypothesis. Political party and economic status are related, but determining how they are related is a limitation of our research. Further work must be done to determine whether it is political leaning that determines economic status or economic status that determines political leaning, or both. Unraveling the relationship between politics and economics can provide insights into the symptoms, causes, and possible solutions to the US’s growing polarization. Keywords: partisanship, household income, homeownership, poverty, population, education