Online Proceedings

Poster Session 1

Commons East, Easel 55

11:00 AM to 1:00 PM

The Evolution of Cooperation - C++ Coding of Social Networks using "The Prisoners Dilemma"

Brandon Berk, Junior, Mathematics, Heritage College McNair Scholar
Mentor: Feng Fu, Dartmouth College

"The Prisoner's Dilemma" (PD) is a Game Theory activity in which a group of individuals simultaneously decide between two choices without knowledge of each other's selection and where individual profit depends on the responses of all participants. PD, when graphed, is often used to mathematically model human cooperation and social interaction. This research delves into the game played repetitively while looking to determine interactions between cooperation and defection in terms of maximal profit. The game was coded using C++ to visually and analytically depict cooperation and defection. A simple lattice structure graph with periodic boundary conditions was used, thus each node had four connections. When running the model, it became evident that defection dominated the game. For cooperation to remain in the game, very strict conditions must be met. As PD in game theory is very basic but structured, it has been used to model a wide range of phenomena in a multitude of disciplines such as stock market changes, bacteria growth, and the decision making of world leaders. The model created in this research is very simple, but with more parameters and variations to the model, we hope to create a more complex system that is capable of modeling such things as social networks as well as take into account socioeconomic factors to model true human cooperation statistics.

Session 1B

Data Science, Statistics and Society

Session Moderator: , Statistics and Sociology

MGH 082A

12:30 PM to 2:15 PM

* Note: Titles in order of presentation.

Monte Carlo Simulation Estimations of π

Samuel (Sam) Wolf, Sophomore, Computer Science , Mathematics , Lake Wash Tech Coll
Mentor: Narayani Choudhury, Math, Lake Washington Institute of Technology

Monte Carlo simulations employ random probability distribution statistics to estimate areas and volumes. Here, we employ Monte Carlo simulations to estimate the numerical value of π. We inscribe a circle in a square board and throw N darts using random values for both x and y. The probability that the dart lies within the circle = area of circle/area of square. This relationship allows us to estimate π. We wrote EXCEL/JAVA code for this research. The accuracy of estimated π is improved as the number of darts N –> ∞. This research allows us to combine mathematics, computer programming and data visualization to estimate π. Important applications of Monte Carlo simulations to find areas and volumes of complex objects including rivers, landscapes and organisms which cannot be represented by analytic functions will be discussed.

Session 1B

Data Science, Statistics and Society

Session Moderator: , Statistics and Sociology

MGH 082A

12:30 PM to 2:15 PM

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Multivariable Calculus Applications in Environmental Sciences

Morgan Wolf, Freshman, Math, Physics , Lake Wash Tech Coll
Samuel (Sam) Wolf, Sophomore, Computer Science , Mathematics , Lake Wash Tech Coll
Mentor: Narayani Choudhury, Math, Lake Washington Institute of Technology

Here we explore applications of multivariable calculus for studying three dimensional wave media in our environment. We employ regression based methods to derive analytic formulae for real wave. Using multivariable optimization methods, we derive the maxima, minima and saddle points of three dimensional functions. We use advanced data visualization methods to study the divergence and curl and illustrate how these can be used to study ocean waves- including their vorticity and circulation. The project provides hands on exploration of real world environmental science problems with advanced data visualization and shows how divergence and curl can be used to measure circulation and vorticity parameters of real wave media involving ocean waves. Real world manifestations of scalar and vector fields in our environment are also presented.

POSTER SESSION 2

Balcony, Easel 110

1:00 PM to 2:30 PM

Uniformity of Solutions to Diophantine Equations

Rohan Koosha Hiatt, Senior, English, Mathematics
UW Honors Program
Daria Micovic, Senior, Mathematics
Bryan Tun Pey (Bryan) Quah, Senior, Mathematics
Blanca Vina Patino, Senior, Mathematics
Mentor: Amos Turchet, Mathematics
Mentor: Travis Scholl, Mathematics

Diophantine equations are polynomial equations with integer coefficients. In 1970, Matiyasevich proved that an algorithm to decide whether a given Diophantine equation has a solution does not exist. We investigated some such equations including Pell and Thue equations, that have an infinite and finite number of solutions, respectively. Geometrically the solution set of Diophantine equations in 2 variables corresponds to plane curves, e.g. elliptic or hyperelliptic curves, and the number of rational solutions is related to the genus of the corresponding curve. We gathered statistics on the size of the solution sets in an attempt to address an important unsolved problem in arithmetic geometry, known as Uniformity Conjecture of Caporaso, Harris and Mazur. Our project involved an analysis of Python and C code, specifically through the CoCalc development platform, utilizing data from the L-functions and Modular Forms Database (LMFDB). Currently our results agree with the current known data on the sizes of solution sets, and we hope to extend our results to gather data on curves not currently in the LMFDB. This will serve as a testing ground for the Uniformity Conjecture.

POSTER SESSION 2

Balcony, Easel 113

1:00 PM to 2:30 PM

WXML: Randomly Mixing Fluids

Keith Michael Fife, Senior, Mathematics (Comprehensive)
Mentor: Soumik Pal, Mathematics
Mentor: Max Goering, Mathematics

Fluid mixing is big business, both its application and research. Many manufacturers must be able to reliably mix fluids to a satisfiable level of homogeneity. Fluids taking longer to mix are fluids not being used for their intended purpose. This research explores two specific uses of random processes in an attempt to increase the speed and/or quality of the mixing process. While most mixing processes are in practice deterministic, the ansatz of this inquiry is that fluid mixing may be better applied through randomness. In this research, two different models for fluid mixing are explored, implemented and statistical measurements are taken. The first is the Gather and Spread model proposed by Diaconis and Pal in 2017, the second is the Toroidal Lattice Mixing model proposed by Pierrehumbert in 2000. We calculate Kendall's τ (tau), and show evidence for a linear relationship between the number of points in the Gather and Spread model, and the number of trials. We show evidence against Gather and Spread distributing its points in Poisson manner. We calculate the standard deviation and relative entropy in the Toroidal Lattice Mixing model. We find strong evidence for the TLM having an exponential mixing rate, and strong evidence suggesting that advection diffusion mixing methods are always faster than diffusion alone.

POSTER SESSION 2

Balcony, Easel 114

1:00 PM to 2:30 PM

The Combinatorics of Factorial Base Representation

Tina Rajabi, Sophomore, Mathematics, Edmonds Community College
Mentor: Tom Edgar, Mathematics, Pacific Lutheran University

Every non-negative integer can be written using what is known as the factorial base representation. We define this notion and explore certain combinatorial structures arising from the arithmetic of these representations. In particular, we investigate the sum-of-digits function, carry sequences, and a partial order referred to as digital dominance. Finally, we describe an analog of a classical theorem due to Kummer that relates the combinatorial objects of interest by constructing a variety of new integer sequences. Many of the combinatorial objects we describe are of interest in their own right and provide interesting avenues for future discoveries. For instance, the carry sequences we describe correspond to a new family of integer partitions that are analogous to hyper-binary partitions, which have been extensively studied. Moreover, the digital dominance order most likely contains interesting combinatorial information about the integers as represented in the factorial base. Finally, our analog of Kummer's theorem has helped to explain how the classical version of Kummer's theorem can be altered to fit with almost any positional numeration system.

Poster Session 2

Balcony, Easel 112

1:00 PM to 2:30 PM

Counting Densities of Discrete Sets

Kimberly B. Bautista, Senior, Mathematics
Madeline E. (Maddy) Brown, Senior, Physics: Comprehensive Physics, Mathematics
UW Honors Program
Pilhyun Andrew (Andrew) Lim, Junior, Mathematics
Mentor: Jayadev Athreya, Mathematics and CHID
Mentor: Samantha Fairchild, Mathematics

If you're tiling a circular room with square tiles, how many do you need? What if your tiles are parallelograms? As the circle gets larger, is there a pattern? Mathematically, this can be framed as counting integer points in a large circle. We're interested in how this count changes as you change the shape of the tiles, and to understand this better, we started by counting the number of integer vector pairs within a ball of radius R so that the parallelogram they make has a fixed area (determinant). We created a Python program that would generate all of the primitive points out to radius R, which we then extended to count the number of vector pairs with a certain determinant k. We were able to compute the limiting density of this count, extending known results for the case of determinant 1. We are now studying other discrete sets, such as ones generated by objects from hyperbolic geometry, known as Hecke triangle groups. In the future, we plan to use our research for counting pairs of vectors in different spaces and generalize it for counts of k-tuple vectors. Come for floor tilings, stay for beautiful pictures!

Poster Session 2

Balcony, Easel 111

1:00 PM to 2:30 PM

The Mathematics of Gerrymandering

Leon Luca (Leo) Segovia, Senior, Economics, Mathematics
Weifan Jiang, Senior, Applied & Computational Mathematical Sciences (Discrete Mathematics & Algorithms), Computer Science
Namyoung Kim, Junior, Exchange - Arts & Sciences
Alexander Michael (Alex) Robkin, Senior, Mathematics (Comprehensive)
Mentor: Christopher Hoffman, mathematics
Mentor: Tejas Devanur, Mathematics

This research focuses on the mathematics of gerrymandering. Gerrymandering refers to how political parties draw district boundaries to give them better odds at receiving a majority during congressional elections. Every 10 years district boundaries in the United States are redrawn for congressional elections by the current majority party. With such change in political cartography, redrawing district boundaries not only affects who wins but also the people who live within them. Given the loosely defined criteria for redistricting, we seek to quantify the process of redrawing district lines to determine when this partisan process becomes an act of manipulation and exclusion. We will be use The Metropolis Hastings (M.H.) Algorithm to produce a random walk that has a stationary distribution that will be the probability distribution of all possible districting plans for a given U.S state. By collecting congressional maps and election results, we implemented the M.H. algorithm in python to produce sample spaces for Washington State and Iowa. We will simulate local and federal redistricting requirements using self-defined parameters which measure: compactness of districts, population division, division of counties, and minority voter populations. We simulated election results with respect to our sample districts and compared the simulated outcomes with actual district maps and their election results. This study helps to determine if current redrawn maps legitimately reflect the people they purport to represent or if they are manipulations and subversions of our democratic process.

Session 2Q

Astronomy and Engineering

Session Moderator: Suzanne Hawley, Astronomy

JHN 026

3:30 PM to 5:15 PM

* Note: Titles in order of presentation.

Vector Calculus Studies of Fullerenes

Iuliia Dmitrieva, Freshman, Computer and Electrical Engineering, Lake Wash Tech Coll
Rami Manad, Sophomore, Mechanical Engineering, Aerospace Engineering, Lake Wash Tech Coll
Tom Skoczylas, Sophomore, Mechanical Engineering, Lake Wash Tech Coll
Mentor: Narayani Choudhury, Math, Lake Washington Institute of Technology

Polymer based fullerenes are used as photovoltaics in solar panels. Fullerene C60 molecules have icosahedral based structures resembling geodesic domes. Fullerenes have convex polyhedral shapes which obey Euler's topological rules. Their novel structures involve Golden ratios. Here we use vector calculus methods to calculate bond lengths and bond angles and provide estimates for the volume and surface area of the molecule. The calculated average bond length (1.4320 Å), edge length (2.4252 Å), bond angle (116o) are in good agreement with reported experiments. The estimated fullerene molecularvolume is 788 Å3 and surface area is 426 Å2. To understand the critical effect of dimensionality on

volume, we have studied the volume of a hypersphere in n-dimensions. The project provides hands on exploration of real world problems and data visualization.