Session 1R

Computer Science: Distributed Systems, Verification, Security and HCI

Session Moderator: Kurtis Heimerl, Computer Science and Engineering

JHN 111

12:30 PM to 2:15 PM

* Note: Titles in order of presentation.

Relating Dice to Voting Systems

Arthur Vartanyan, Senior, Mathematics
Satvik Agarwal, Sophomore, Computer Science (Data Science)
Jueyi Liu, Senior, Applied & Computational Mathematical Sciences (Scientific Computing & Numerical Algorithms)
Dorothy Truong, Junior, Applied & Computational Mathematical Sciences (Discrete Mathematics & Algorithms)
Mentor: Jonah Ostroff, Mathematics
Mentor: Lucas Van Meter, Mathematics

For our research, we are focusing on dice and how we can relate them to voting systems. We want to see if we can create dice systems that mimic voting systems (with >2 candidates) and the traits that the systems possess. Two ideal traits of these voting systems are Pareto, which means that if everyone likes candidate A more than B, candidate B should lose; and IIA, which is if candidate A wins, then everyone mixes their ballot but keeps A's rank relative to B the same, then A should still win. However there is Arrow's Theorem, a central focus of our research, which states that if your system is Pareto and IIA, you must live in a dictatorship. One major topic of our research is trying to show whether or not this holds for our die systems. In order to accomplish this, we set out to define what these traits would be in terms of dice and how to translate dice outcomes to voter results, which we have successfully done for Pareto and IIA thus far. Our methods for this included testing certain systems that would reflect moving candidates/voters around, and testing these systems using some code we wrote for this purpose. Some other topics for our research are translating specific voting systems, such as popular vote, pairwise competitions, and point systems to dice, and showing whether or not these ideal traits hold for these translations, which we have successfully done for some. With this, we hope to gain a better understanding of these systems and how using something truly random, such as dice, has any reflection on the systems we have in place.

Poster Session 2

Balcony, Easel 111

1:00 PM to 2:30 PM

Hausdorff Dimension of Random Curves

Alex Forney, Senior, Mathematics (Comprehensive), Applied & Computational Mathematical Sciences (Discrete Mathematics & Algorithms)
Xiyi Yan, Senior, Economics, Mathematics
Ran Zhao, Senior, Mathematics
Zach Dingels, Senior, Mathematics
Mentor: Krzysztof Burdzy, Mathematics
Mentor: Clayton Barnes

People have an instinctive understanding of what it means for an object to be three-dimensional: it has height, width, and length. There is similar familiarity with two-dimensional shapes like squares and triangles, as well as one-dimensional shapes such as lines. The dimensions of some objects, on the other hand, cannot be understood in this familiar way. Take, for example, a set of points in the xy-plane that do not form a straight line or any familiar shape from geometry. One way to measure the dimension of these objects is to calculate the Hausdorff dimension of the set of underlying points. Intuitively, the Hausdorff dimension is a measure of how many small boxes it would take to cover an object. Our research aimed to calculate the Hausdorff dimensions of more complicated curves, such as those arising from Brownian motion, which is a probabilistic model for a randomly moving particle. In particular, we set out to corroborate known results about the Hausdorff dimensions of Brownian motion and the Brownian frontier. Further, we wanted to strengthen unsolved conjectures regarding the Hausdorff dimension of the Brownian earthworm model, which is a model of mass redistribution on the d-dimensional integer lattice introduced by Professor Krzysztof Burdzy. We attempted to develop answers to these problems by simulating Brownian motion in two dimensions and performing statistical analysis on our results. Our model correctly calculated the known dimensions and was in line with Professor Burdzy's previous conjecture regarding the

Brownian earthworm. It is our goal that this research will ultimately contribute to a greater understanding of the earthworm model.

POSTER SESSION 2

Balcony, Easel 108

1:00 PM to 2:30 PM

AKS Primality Test: A Deterministic Primality Algorithm in Polynomial Time

Daria Micovic, Senior, Mathematics
Blanca Vina Patino, Junior, Pre-Major (Arts & Sciences)
Bryan Tun Pey (Bryan) Quah, Junior, Pre-Major (Arts & Sciences)
Rohan Koosha Hiatt, Junior, Mathematics
Mentor: Amos Turchet, Mathematics
Mentor: Travis Scholl, Mathematics

Prime numbers are fascinating objects in mathematics, fundamental to number theory and cryptography in particular. A primality test takes an integer as input and outputs whether that number is prime or composite. Most practical applications require primality tests to be efficient. Today, the largest known prime number has over twenty million digits. Proving that a number of this size is prime can be computationally expensive. Most tests are probabilistic and do not guarantee that any given prime number is in fact prime. Other tests fail to filter pseudo-primes, like Carmichael Numbers, that are in fact composite integers. In 2002, Agrawal, Kayal, and Saxena published the first deterministic primality test that also runs in polynomial time relative to the binary representation of the input. Although their algorithm represents an important breakthrough in the field of computational number theory, it is seldom used in practice. Our objective was to determine why this idealized algorithm is not practical enough compared to other primality tests. We have re-created the algorithm and optimized our initial naïve implementation by studying each step to achieve optimal complexity using Fermat's Little Theorem, modular arithmetic, and the Fast Fourier Transform for multiplicative efficiency. To confirm that probabilistic methods are still preferred, we compared execution times and accuracy of other tests to our own results.

POSTER SESSION 2

Balcony, Easel 110

1:00 PM to 2:30 PM

Nontransitive Dice and Social Choice

Jacob Adam (Jacob) Watkins, Senior, Mathematics, Physics: Comprehensive Physics
Robert Murray (Robert) Gunn, Senior, Physics: Comprehensive Physics, Mathematics
Jase Grills, Senior, Applied & Computational Mathematical Sciences (Discrete Mathematics & Algorithms)
Mentor: Jonah Ostroff, Mathematics
Mentor: Lucas Van Meter, Mathematics

The purpose of this research is to better understand the connections between dice and elections in voting theory. We explored the phenomenon of nontransitivity in dice and elections. In a set of 3 dice, labeled A, B, C, it is straightforward to designate sides such that, on average, A beats B, B beats C, and C beats A. Similarly, in an election of more than two candidates, where voters create a preference list for the set of candidates, it is simple to make nontransitive relationships in voter preference. Motivated by these situations, we aimed to find a correspondence between these two mathematical objects. Additionally, we defined the concept of an overall winner for a set of dice and an election, and explored the effects of nontransitivity in determining it. We used tournaments (from graph theory) as a tool to visualize the relationship structure among dice and elections. As a result of this work, we have shown that, from any dice set, an election can be constructed such that the dice possess the same winning-losing relationships. The converse is also true: given an election, a set of dice can be constructed with identical winning structure to the election. We defined the notion of a contest, which serves as a classification for both objects. Finally, we developed a triangular inequality for contests, and proved that dice and elections both satisfy this inequality.

POSTER SESSION 2

Balcony, Easel 109

1:00 PM to 2:30 PM

Trees, Forests, and Entropy

Melissa Maria Stadt, Senior, Mathematics
William Lewis (Will) Dana, Senior, Mathematics (Comprehensive)
UW Honors Program
Lalith Manohar Reddy (Lalith) Devireddy, Senior, Mathematics (Comprehensive)
Xinwei (Vivian) Fan, Senior, Applied & Computational Mathematical Sciences (Discrete Mathematics & Algorithms)
Mentor: Doug Lind, Mathematics
Mentor: James Morrow, Mathematics
Mentor: Samantha Fairchild, Mathematics

A common class of problems is that of understanding the large-scale properties of a system defined by small-scale relationships. These problems arise in fields such as statistical mechanics. We have been considering such problems as they apply to graphs, mathematical structures that can be used to represent networks, by investigating the limiting behavior of finite approximations to infinite graphs. In particular, we have been studying the growth rate of the number of spanning trees on these finite graphs. This allows us to quantify the complexity of various infinite graphs. The Matrix-Tree Theorem gives a formula for the number of spanning trees on a finite graph using techniques from linear algebra. This allows us to numerically approximate growth rates. These calculations can help us see patterns and develop theoretical approaches for obtaining exact values, as well as test theoretical results. In the case of graphs with translational symmetry, we have used techniques from group theory, a powerful area of mathematics, to derive an elegant expression for the exact growth rate in terms of Mahler measure. This is an integral formula connected to dynamical systems and number theory, among other research areas.While these considerations on spanning trees are facilitated by the Matrix-Tree Theorem, there is no known formula for counting spanning forests, a structure consisting of one or multiple trees which is more natural in some ways. Our current research generalizes the questions on trees to various types of spanning forests. We have also been exploring different ways to consider how a graph grows. This work can be used to build on this active research area of determining properties of large-scale structures.

POSTER SESSION 2

Commons East, Easel 77

1:00 PM to 2:30 PM

Gravitational Billiards

Nvida Ange Janine (N'vida) Yotcho, Junior,
Mentor: Jayadev Athreya, Mathematics and CHID

For many, mathematics is just the language of numbers. But, what people forget is that the universe is made of numbers, thus mathematics is its language. That is why we used a mathematical approach to study the long-term behavior of a moving ball influenced by the Earth's gravitational field. For our purpose, we experimentally defined gravitational billiards to be vertical billiards in which the earth gravitational field affects the billiards ball's motion. We also assumed that the billiards boundary could be any mathematical function of our choice, and the experimental system was conservative. Once in motion, the billiards ball followed a two-dimensional projectile trajectory until collision with the boundary occurred. After that, the ball engaged in a new projectile trajectory. As a matter of fact, each trajectory between two points of ball-boundary collision could be considered as an isolated two-dimensional gravitational motion with its own set of conditions. We took advantage of that aspect to build different simulations in parabolic, circular, paraboloidal and spherical gravitational billiards. The long-term behavior of the ball under gravity in our different gravitational billiards presented some aspects that so far corroborated the physics behind our study.