

# Undergraduate Research Symposium May 18, 2012 Mary Gates Hall

## Online Proceedings

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### POSTER SESSION 1

MGH 241, Easel 175

12:00 PM to 1:30 PM

#### Analyzing and Demonstrating the Robustness of Self-Stabilizing Inverted Pendulums

*Jared Szi, Junior, Mathematics, University of California, Davis*

*McNair Scholar*

*Mentor: Dan Romik, University of California, Davis*

Many examples of control systems are found in modern-day industrial applications, ranging from the Segway to cruise control in an automobile. The self-stabilizing inverted pendulum serves as an archetypal problem by which the robustness of such control systems, which use feedback to control themselves, is measured. In this project, I analyze several simple control systems involving the inverted pendulum, and attempt to find their stability solution. I have also built a computer simulation in Mathematica, a computational software package, which demonstrates fundamental concepts in control theory, as well as the difficulty of manual control and the usefulness of feedback in self-stabilizing systems. This simulation provides a useful learning tool to users who otherwise have little experience with the theory of control. Finally, I investigate theoretical problems related to the so-called radius of stability, which is a number that tells us how far we may deviate from equilibrium before the control system fails. I hypothesize that as further variables are introduced into a control model, the radius of stability will decrease accordingly. Since control systems are widely used in modern day society, it is important that we understand the stability of such systems, thereby maximizing the safety and effectiveness of their applications.

#### The Ihara Zeta Function on Graphs

*Elizabeth Landicho (Elizabeth) Wicks, Senior, Mathematics (Comprehensive), Physics*

*Mary Gates Scholar*

*Mentor: Ralph Greenberg, Mathematics*

The Ihara Zeta Function (IZF) of a finite graph encodes its spectral, combinatorial and number theoretic properties. Our aim is to calculate the IZF of the Cayley graph of the modular group  $PSL(2, \mathbb{Z})$ , which can be thought of as a special group of square matrices. Since this group plays a central role in number theory and other areas of mathematics and physics, we hypothesize that the zeta function will reflect important information about modular forms and elliptic curves. Our previous research has yielded an algorithm that computes the IZF of a sequence of finite approximations of this group, but it does not give the IZF of the entire graph. The IZF can be expressed as a product of L-functions, which encode how successive graphs in the sequence are related. Our ultimate aim is to use the L-functions of the finite approximations to compute the IZF of the modular group. We are currently exploring L-functions of simpler Cayley graphs, in the hope that the methods can be generalized to calculate the IZF of  $PSL(2, \mathbb{Z})$ .

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### SESSION 2M

#### MODERN QUANTITATIVE METHODS FOR REAL WORLD PROBLEMS

*Session Moderator: J. Nathan Kutz, Applied Mathematics*

**Mary Gates Hall 288**

3:30 PM to 5:00 PM

\* Note: Titles in order of presentation.